Enrollment No: _____ Exam Seat No: _____

C. U. SHAH UNIVERSITY

Winter Examination-2021

Subject Name: Transform Methods

Subject Code: 4SC05TRM1 **Branch: B.Sc. (Mathematics)**

Semester: 5 **Date: 15/12/2021** **Time: 11:00 To 02:00** **Marks: 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

(a) Inverse Laplace transform of $\frac{12}{s^2-9}$. 01

(a) $3\sinh 4t$ (b) $4\sinh 3t$ (c) $4\cosh 3t$ (d) $3\cosh 4t$

(b) $L(4^t) =$ ____. 01

(a) $\frac{1}{s-ln4}$ (b) $\frac{1}{s+ln4}$ (c) $\frac{s}{s-ln4}$ (d) None

(c) Laplace transform of $\cos at$ is given by_____. 01

(a) $\frac{s}{s^2+a^2}$ (b) $\frac{s}{s^2-a^2}$ (c) $\frac{1}{s^2+a^2}$ (d) $\frac{1}{s^2-a^2}$

(d) If $f(x) = x$ in $(-\pi, \pi)$ then Fourier series co-efficient $a_0 =$ ____. 01

(a) 1 (b) 0 (c) π (d) None

(e) $Z(a^n) =$ _____. 01

(a) $\frac{z}{z-a}$ (b) $\frac{z}{z+a}$ (c) 1 (d) None

(f) The period of $\sin nx$ is_____. 01

(a) $\frac{2\pi}{n}$ (b) 2π (c) $\frac{\pi}{2}$ (d) None

(g) If $f(x) = \begin{cases} -k\,; -\pi < x < 0 \\ k\,; \quad 0 < x < \pi \end{cases}$ and $f(x + 2\pi) = f(x)$ then $f(x)$ is an____. 01

(a) even function (b) odd function (c) neither odd nor even
(d) None

(h) Write the Dirichlet's condition for the Fourier series. 02

(i) State the Damping rule for Z- transform. 02

(j) $L\left\{\frac{f(t)}{t}\right\} =$ _____. 01

(k) Define: Fourier Cosine transform. 01

(l) If $x = c$ is a point of discontinuity then the Fourier series of $f(x)$ at $x = c$ 01

gives $f(x) =$ _____.

Attempt any four questions from the Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

(a) Show that $x^2 = \frac{\pi^3}{3} + \sum_{n=1}^{\infty} \frac{(-1)^n cosnx}{n^2}$ in the interval $(-\pi, \pi)$. 05

(b) Find the Fourier series of $f(x) = x$, where $-2 < x < 2$. 05

(c) Find $L^{-1}\left\{\frac{(\sqrt{s}-1)^2}{s^3}\right\}$. 04

Q-3 **Attempt all questions** **(14)**

(a) Find $L^{-1}\left(\frac{2s^2-6s+5}{s^3-6s^2+11s+6}\right)$. 05

(b) Find $L(t\, e^{-2t} \cos ht)$. 05

(c) Find half range cosine series of $f(x) = \pi - x, 0 < x < \pi$. 04

Q-4 **Attempt all questions** **(14)**

(a) State and prove Euler's formulae for Fourier series. 07

(b) State and prove convolution theorem and hence evaluate using it to find $L^{-1}\left\{\frac{1}{s(s-2)}\right\}$. 07

Q-5 **Attempt all questions** **(14)**

(a) Find Fourier integral representation of the function, 07

$f(x) = \begin{cases} 1\,; if\ |x| < 1 \\ 0\ ;\ if |x| > 1 \end{cases}$ hence evaluate $(i) \int_0^{\infty} \frac{sin\lambda\ cos\lambda x}{\lambda}\, d\lambda (ii) \int_0^{\infty} \frac{\sin\lambda}{\lambda}\, d\lambda$

(b) Expand the function $f(x) = x \sin x$ as a Fourier series in the interval $0 \le x \le 2\pi$. 07

Q-6 **Attempt all questions** **(14)**

(a) Solve the equation $\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 5x = e^{-t} \sin t,\ x(0) = 0, x'(0) = 1$ by using Laplace transform. 07

(b) Find Fourier transform of $f(x) = e^{-ax^2}$ and hence deduce that 07

$$F\left(e^{-\frac{x^2}{2}}\right) = e^{-\left(\frac{\lambda^2}{2}\right)}.$$

Q-7 **Attempt all questions** **(14)**

(a) Find the Fourier sine and cosine transform of $f(x) = e^{-ax}, a > 0$. 05

(b) Find a Fourier series of $f(x) = 1 + \sin x$ in the interval $-1 < x < 1$. 05

(c) If $F(\lambda)$ is the Fourier transform of $f(x)$ then $F\big(f(x-a)\big) = e^{-i\lambda a} F(\lambda)$. 04

Q-8 **Attempt all questions** **(14)**

(a) Find $L\{f'(t)\}$ if $f(t) = \begin{cases} 2t\,; 0 \le t \le 1 \\ t\ ;\quad t > 1 \end{cases}$. 05

(b) If $U(z) = \frac{2z^2+3z+4}{(z-3)^3}$ then find u_0, u_1 and u_2. 05

(c) Prove that $Z(\sin n\theta) = \frac{z \sin \theta}{z^2-2z \cos \theta+1}$. 04